一般的波动理论

波函数

基本概念

一种(广义)的物理场的时空分布

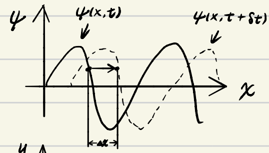
具体例子

简谐波(1D)

$\psi(x,t) = \vec{A}e^{i(kx-wt)} = \vec{A}e^{i\varphi}$

相速度 $= \frac{\text{位移}}{\text{时间}}$ $\vec{v} = \frac{w}{|k|}\hat{k}$

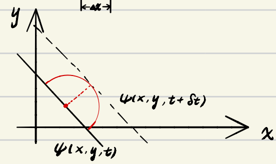
$v = \left(\frac{\partial x}{\partial t}\right)_\varphi = \frac{-(\partial\varphi/\partial t)_x}{(\partial\varphi/\partial x)_t} = \frac{w}{k}$



简谐波(2D)

$\psi(x,y,t) = \vec{A}e^{i(k_x x+k_y y-wt)}$

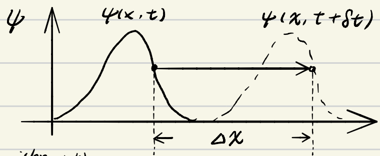
$v = \frac{w}{\sqrt{k_x^2+k_y^2}}$



脉冲波(1D)

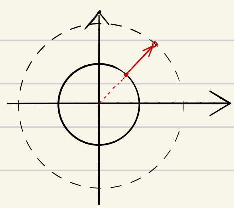
$\psi(x,t) = \vec{A}\frac{1}{(x-ut)^2+1}$

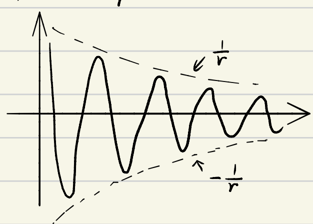
$v = u$



球面波(3D)

$\psi(x,y,z,t) = \psi(r,\theta,\phi,t) = \frac{\vec{A}}{r}e^{i(kr-wt)}$





能量守恒

$\rho_E V = \rho_E \cdot 4\pi r^2 \delta r = Const$

$\rho_E \propto |\psi(x,t)|^2$

$\frac{|\psi(x,t+\Delta t)|^2}{|\psi(x,t)|^2} = \frac{R^2(t)}{R^2(t+\Delta t)}$

$|\psi(x,t)| \propto \frac{1}{R}$

$$\begin{cases} \nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \\ \nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) = \frac{\partial^2\psi}{\partial r^2} + \frac{2}{r}\frac{\partial\psi}{\partial r} = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) \end{cases}$$

$$\Rightarrow \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \Rightarrow \frac{\partial^2}{\partial r^2}(r\psi) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}(r\psi)$$

$$\Rightarrow r\psi(\vec{r},t) = Ae^{i(\vec{k}\cdot\vec{r}-wt)} \quad \Rightarrow \psi(\vec{r},t) = \frac{A}{r}e^{i(\vec{k}\cdot\vec{r}-wt)}$$

波动方程

推导

①寻找对偶场 $\vec{A}$ v.s. $\vec{B}$

②建立耦合关系 $\partial_t \vec{A} = \alpha \partial_x \vec{B}$ $\partial_t \vec{B} = \beta \partial_x \vec{A}$ 时间偏导、空间偏导

③解耦合 $\partial_t^2 \vec{A} = \alpha \partial_x \partial_t \vec{B} = \alpha\beta \partial_x^2 \vec{A}$

④波源 $\underbrace{\partial_t^2 \psi(x,t)}_{\text{加速度}} = v^2 \partial_x^2 \psi(x,t) + \underbrace{f(t)\delta(x-x_0)}_{S(x,t)}$ 驱动力

求解 (以1D简谐波为例)

$\partial_t^2 \psi(x,t) = v^2 \partial_x^2 \psi(x,t)$

通解 $\psi(x,t) = f(x+vt) + g(x-vt)$

特解 $\psi(x,t) = \vec{A} e^{i(kx-wt)}$

参数制约

波动方程的形式决定了波函数的形式

波动方程的参数决定了波函数的参数

特例

1D简谐波

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \cancel{\frac{\partial^2}{\partial y^2}} + \cancel{\frac{\partial^2}{\partial z^2}}$$

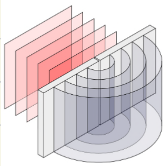
$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \Rightarrow \psi(x,t) = \vec{A} e^{i(kx-wt)}$$

球面

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \cancel{\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial \theta}\right)} + \cancel{\frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \varphi^2}}$$

$$\frac{\partial^2}{\partial r^2}(r\psi) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}(r\psi) \Rightarrow \psi(r,t) = \frac{\vec{A}}{r} e^{i(\vec{k}\cdot\vec{r}-wt)}$$

柱面



$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}} + \cancel{\frac{\partial^2}{\partial z^2}}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \qquad \psi(r,t) = \varphi(r)T(t)$$

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{d\varphi(r)}{dr}\right] - \lambda\varphi(r) = 0 \quad \text{Bessel 方程}$$

$$\psi(r,t) \approx \frac{\vec{A}}{\sqrt{r}} e^{i(\vec{k}\cdot\vec{r}-wt)}$$

电磁波

波函数

光学只研究平面波

$$\begin{cases} \vec{E}(\vec{r},t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \\ \vec{B}(\vec{r},t) = \vec{B}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \end{cases}$$

参数制约关系

波函数参数 $(\mu,\varepsilon) \Rightarrow (\vec{E}_0, \vec{B}_0, \vec{k}, \omega) = (|E_0|, \hat{E}_0, |B_0|, \hat{B}_0, |k|, \hat{k}, \omega)$

	M1	M2	M3	M4
$\vec{E}(\vec{r},t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$	$\hat{E}_0\cdot\hat{k} = 0$ $\vec{\nabla}\cdot\vec{E}_0 = 0$		方向：$\hat{k}\times\hat{E}_0 = -\hat{B}_0$ 大小：$\lvert k\rvert\lvert E_0\rvert = \omega\lvert B_0\rvert$	$\hat{k}\times\hat{B}_0 = \hat{E}_0$ $\Rightarrow \frac{\omega}{\lvert k\rvert} = \frac{1}{\sqrt{\mu\varepsilon}} = c$
$\vec{B}(\vec{r},t) = \vec{B}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$		$\hat{B}_0\cdot\hat{k} = 0$ $\vec{\nabla}\cdot\vec{B} = 0$	$\lvert k\rvert\lvert B_0\rvert = \mu\varepsilon\omega\lvert E_0\rvert$ $\vec{\nabla}\times\vec{E} = -\partial_t\vec{B} \Rightarrow$ 方向、大小相同	$\frac{\lvert E_0\rvert}{\lvert B_0\rvert} = \frac{1}{\sqrt{\mu\varepsilon}} = c$

光的物理量

能量密度 w

单位体积的辐射能量

$$w = \frac{\varepsilon}{2}E^2 + \frac{1}{2\mu_0}B^2 \text{（对偶场模方和）} = \varepsilon E^2 = \frac{1}{\mu}B^2$$

能流密度（Poynting矢量）$\vec{S}$ $[\frac{W}{m^2}]$

单位时间通过单位面积传输的能量

$$\vec{S} = w\vec{v}$$

$$\vec{\nabla}\cdot\vec{S} = -\partial_t w$$

= 有在光的情形下等价

$$S = \frac{w\,v\,\Delta t\,A}{\Delta t\,A} = wv$$

$$\vec{S} = \vec{E}\times\vec{H}$$

动量密度 $\vec{p}$

质能关系 $E = mc^2 \Rightarrow w = \rho c^2$

电磁场有质量 $\vec{p} = \rho\vec{v} = \frac{w}{c^2}\vec{v} = \frac{\vec{S}}{c^2} = \frac{1}{c^2}\vec{E}\times\vec{H}$

辐照度 $I = \langle S\rangle_T$

Poynting矢量的时间平均值

$$I = \langle S\rangle_T = \frac{c\varepsilon_0}{2}E_0^2$$

Maxwell's Equation

$$\begin{cases} M1 & \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon} \\ M2 & \vec{\nabla}\cdot\vec{B} = 0 \\ M3 & \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \\ M4 & \vec{\nabla}\times\vec{B} = \mu(\vec{J} + \varepsilon\frac{\partial\vec{E}}{\partial t}) \end{cases}$$

$\Rightarrow$

Electric - Magnetic Equation

$$\begin{cases} \partial_t^2\vec{E} = \frac{1}{\varepsilon\mu}\nabla^2\vec{E} \\ \partial_t^2\vec{B} = \frac{1}{\varepsilon\mu}\nabla^2\vec{B} \end{cases}$$

波动方程

- 对偶场

电场 v.s. 磁场

- 耦合关系

Maxwell's equation ($\rho = 0$, $\vec{J} = 0$)

$$\vec{\nabla} \times \vec{E} = -\partial_t \vec{B} \qquad \vec{\nabla} \times \vec{B} = \mu\varepsilon \partial_t \vec{E}$$

$$\vec{\nabla} \cdot \vec{E} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0$$

- 解耦合

$$\begin{cases} \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\nabla^2 \vec{E} = -\partial_t (\vec{\nabla} \times \vec{B}) = -\mu\varepsilon \partial_t^2 \vec{E} \\ \vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{B}) - \nabla^2 \vec{B} = -\nabla^2 \vec{B} = \mu\varepsilon \partial_t (\nabla \times \vec{E}) = -\mu\varepsilon \partial_t^2 \vec{B} \end{cases}$$

$$\Rightarrow \quad \nabla^2 \vec{E} = \mu\varepsilon \partial_t^2 \vec{E} \qquad \nabla^2 \vec{B} = \mu\varepsilon \partial_t^2 \vec{B}$$

$$\nabla^2 \vec{E} = \frac{1}{v^2} \partial_t^2 \vec{E} \quad \Rightarrow \quad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

- 波源

辐射

光的性质

辐射：光是怎么产生的

辐射

加速运动的电荷辐射电磁波

波动方程的波源项 $\quad \partial_t^2\vec{E} = c^2\nabla^2\vec{E} + \text{Source}$

数学描述

波动方程

Maxwell's Equation
$$\begin{cases} \vec\nabla\cdot\vec{E} = \dfrac{\rho}{\varepsilon} & \vec\nabla\cdot\vec{B} = 0 \\ \vec\nabla\times\vec{E} = -\partial_t\vec{B} & \vec\nabla\times\vec{B} = \mu(\vec{J}+\varepsilon\partial_t\vec{E}) \end{cases}$$

$$\vec\nabla\times(\vec\nabla\times\vec{E}) = \vec\nabla(\vec\nabla\cdot\vec{E}) - \nabla^2\vec{E} = \frac{1}{\varepsilon}\nabla\rho - \nabla^2\vec{E}$$
$$= -\partial_t(\vec\nabla\times\vec{B}) = -\mu\partial_t\vec{J} - \mu\varepsilon\partial_t^2\vec{E}$$
$$\Rightarrow \boxed{\partial_t^2\vec{E} = c^2\nabla^2\vec{E} - \frac{1}{\varepsilon}(\partial_t\vec{J} + c^2\nabla\rho)}$$

电流密度 $\vec{J} = ne\vec{v}$ $\quad \vec{J} = nq\underbrace{\dot{\vec{v}}}_{\text{加速度}} + 0$

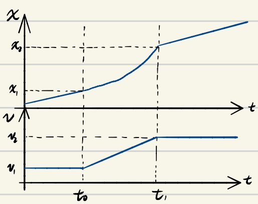
系统

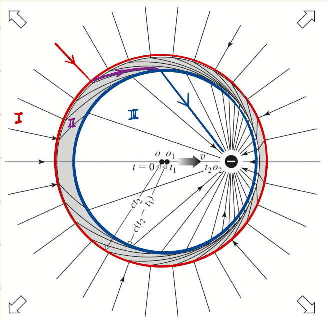
① 加速运动的点电荷
$$\begin{cases} \rho(\vec{r},t) = \delta(\vec{r}-\vec{r}(t)) \\ \vec{J}(\vec{r},t) = \rho q\vec{v} = q\vec{v}(t)\,\delta(\vec{r}-r(t)) \end{cases}$$

② 导线中的加速电流
$$\begin{cases} \rho(\vec{r},t) = 0 \\ \vec{J}(\vec{r},t) = \underbrace{\rho_+ q\vec{v}_+}_{=0} + \rho_- q\vec{v}_- = \rho_- q\vec{v}_- \end{cases}$$

例子（光的偏振、辐照度）

匀加速运动点电荷





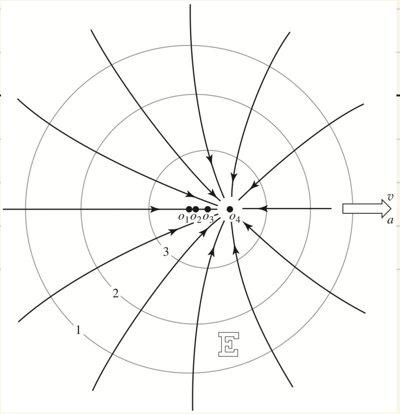
① 电荷对电场的扰动以光速传播

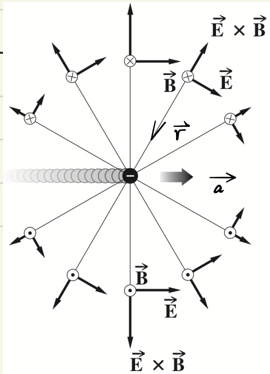
○ 以外的 I 区感受到的是 t_0 时刻前的电场

○ 以内的 II 区感受到的是 t_1 时刻后的电场

② 电场线是连续的

连接 I 区和 III 区对应的电场线得到 II 区





$$\vec{B} \propto \vec{a}\times\vec{r}$$
$$\vec{E} \propto \vec{r}\times(\vec{r}\times\vec{a})$$

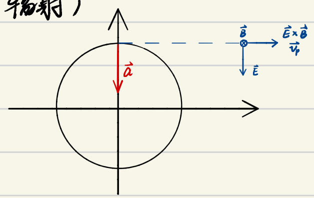
辐照度

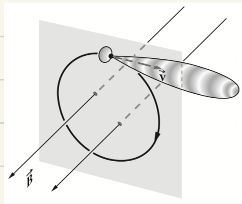
$I(0) = I(180^\circ) = 0$

$I(90^\circ) = I(270^\circ) = I_{max}$

圆周加速运动

（同步辐射）

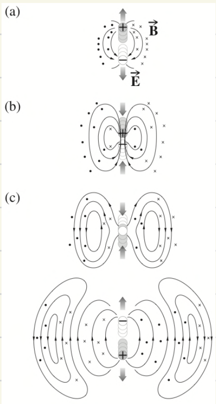




θ：观察点与加速度方向的夹角

$I(\theta)$：$I(0) = I(180^\circ) = 0$ $I(90^\circ) = I(180^\circ) = I_{max}$

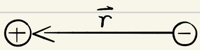
偶极辐射



偶极子

偶极振荡

偶极辐射

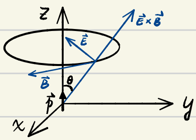


偶极距 $\vec{p} = q\vec{r}$

① 电场线总是从正电荷指向负电荷 或首尾相接成闭环

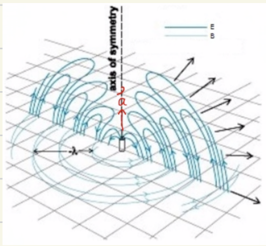
② 电场线不会无缘无故地消失（电场携带能量）

③ 电荷对电场的扰动以光速传播



$\vec{B} \propto \vec{a} \times \vec{r}$

$\vec{E} \propto \vec{r} \times (\vec{r} \times \vec{a})$



远场近似（$kr \gg 1 \Rightarrow r \gg \frac{1}{k}$）

$$E = \frac{p_0 k^2 \sin\theta}{4\pi\varepsilon_0 r} e^{i(kr-\omega t)} \propto \sin\theta$$

$$I(\theta) = \frac{1}{T}\int_0^T S\,dt = \frac{p_0^2 \omega^4}{32\pi^2 c^3 \varepsilon_0} \frac{\sin^2\theta}{r^2} \propto \sin^2\theta$$

设偶极子 $\vec{p} = \vec{p}_0 e^{i(kr-\omega t)}$ $\vec{p}_0 = p_0\cos\theta\,\hat{r} - p_0\sin\theta\,\hat{\theta}$

电场推迟势 $\vec{A} = -\frac{i\mu\omega p_0 e^{i(kr-\omega t)}}{4\pi r}(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})$

$$\vec{B} = \vec{\nabla}\times\vec{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\varphi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ A_r & A_\theta & A_\varphi \end{vmatrix} = \frac{\mu k^2 \omega p_0}{4\pi}\left(\frac{1}{kr} - \frac{i}{k^2r^2}\right)\sin\theta\, e^{i(kr-\omega t)}\,\hat{\varphi}$$

$$\vec{\nabla}\times\vec{H} = \frac{\partial \vec{D}}{\partial t} \Rightarrow \nabla\times\vec{B} = \mu\varepsilon\frac{\partial \vec{E}}{\partial t} = -i\omega\mu\varepsilon\vec{E}$$

$$\vec{E} = -\frac{1}{i\omega\mu\varepsilon}\vec{\nabla}\times\vec{B} = \frac{k^3 p_0}{4\pi\varepsilon}\left\{\left[\frac{2i}{(kr)^2} + \frac{2}{(kr)^3}\right]\cos\theta\,\hat{r} + \left[-\frac{1}{kr} + \frac{i}{(kr)^2} + \frac{1}{(kr)^3}\right]\sin\theta\,\hat{\theta}\right\}e^{i(kr-\omega t)}$$

色散

概念

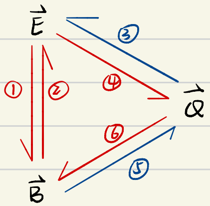
不同颜色的光在介质中发生分散

不同频率的光在介质中折射率不同

介面

波动方程

耦合场



① M4 $\nabla\times\vec B = \mu(\vec J + \varepsilon\frac{\partial \vec E}{\partial t})$

② M3 $\nabla\times\vec E = -\frac{\partial\vec B}{\partial t}$

③ M1 $\nabla\cdot\vec E = \frac{\rho}{\varepsilon}$

④ 库仑定律 $\vec E = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}\hat r$

⑤ 洛仑兹力 $\vec f = \rho\vec E + \vec J\times\vec B$

⑥ M4 $\nabla\times\vec B = \mu(\vec J + \varepsilon\frac{\partial \vec E}{\partial t})$

波动方程

微观世界 (单个电偶极子) 电偶极振荡函数

① 研究对象 x

② 建立方程(驱动) $m_e\partial_t^2 x = -m_e\omega_0^2 x + qE_0\cos\omega t$

③ 求解方程

通解 $x(t)=\alpha\cos\omega_0 t+\alpha'\sin\omega_0 t+\beta\cos\omega t+\beta'\sin\omega t$

特解 { 初始条件 $x(t=0)$ 的位移和速度在物理上不重要 $\alpha=\alpha'=0$

驱动同相位 $\beta'=0$

$\Rightarrow x(t) = \frac{q/m_e}{\omega_0^2-\omega^2}E_0\cos\omega t$

$\vec P = Nq\vec r$

宏观世界 电介质

① 色散来自于折射率 $n = n(\omega)$

② $n = \frac{c}{v} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} \approx \sqrt{\frac{\varepsilon}{\varepsilon_0}}$

③ 介电材料中电偶极子贡献 $E_{dipole}\Rightarrow\varepsilon$

$\vec D = \varepsilon_0\vec E + \vec P = \varepsilon\vec E \Rightarrow \varepsilon = \varepsilon_0 + \frac{P}{E}$

波函数

物理性质

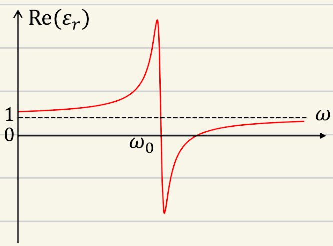
色散

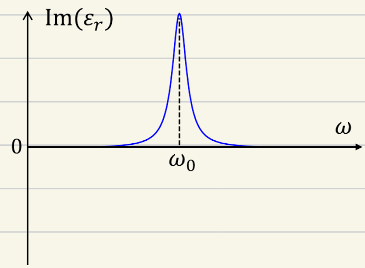
从实验/经验可知 $\psi = E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$$m_e \frac{d^2x}{dt^2} = f + F + F_{dis} = -m_e\omega_0^2 x + q_e E_0 \cos\omega t - m_e\gamma\frac{dx}{dt}$$

$$n^2(\omega) = 1 + \frac{Nq_e^2}{\varepsilon_0 m_e}\left(\frac{1}{\omega_0^2-\omega^2+i\gamma\omega}\right) = 1 + \frac{\beta}{\omega_0^2-\omega^2+i\gamma\omega}$$

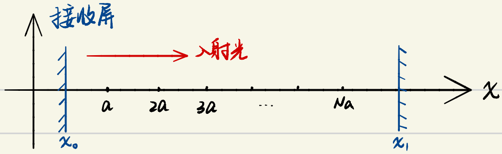
$$Re(n^2) = 1 + \frac{\beta(\omega_0^2-\omega^2)}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2} \qquad Im(n^2) = \frac{-\beta\gamma\omega}{(\omega_0^2-\omega^2)^2-\gamma^2\omega^2}$$





光在介质中的传播：散射 & 干涉

基本物理过程	散射与干涉的竞争与合作
合作	先单偶极子散射再集体干涉
竞争	谁起主导作用 { 介质密度小时，散射主导 / 介质密度大时，干涉主导　判据①
散射	单个偶极子的"受激辐射"
过程	偶极子激发 + 偶极子辐射　$(\omega_i = \omega_{偶极} = \omega_0)$
散射的频率依据	判据②
	偶极激发　$\vec{u} = \frac{\beta \vec{E}}{\omega_0^2 - \omega^2}$
	入射光 ω 越靠近 ω_0，激发越大
	偶极辐射　$I = \frac{p_0^2 \omega^4}{32\pi^2 c^3 \varepsilon_0} \frac{\sin^2\theta}{r^2}$
	入射光 ω 越大，辐射越强
例子	瑞利散射
	稀薄外层大气 $\Rightarrow$ 判据① $\Rightarrow$ 散射主导
	$\omega_{可见光} < \omega_{空气频率}$ $\Rightarrow$ 判据② $\Rightarrow$ 蓝光更散射
	散射体尺度小 $d \sim \frac{\lambda}{15}$
干涉	
过程	多个偶极子的辐射波之间的干涉
判据③	入射光方向决定相长/相消干涉方向
	相长干涉　顺着入射光方向 } 光在介质中总是向前传播
	相消干涉　逆着，侧向 } (尽管有时向后散射，但向后的散射干涉相消)
数学证明	一维系统中的光向前相长，向后相消
	(见下页*)



* 干涉的相长相消方向

系统

入射光波函数 $\vec{E} = \vec{E}_0 e^{i(kx-\omega t)}$

散射体(偶极子)位置 $x_n = na$

动力学过程

光传播

入射光激发偶极振荡 ① ⇒ 偶极辐射出次级波 ② ⇒ 各次级波在接收屏上干涉 ③

①偶极振荡

u_n：第 n 个偶极子偏离平衡位置的位移 $\vec{p} = q\vec{u}_n$

动力学方程：$m\ddot{u}_n = -m\omega_0^2 u_n + \underbrace{\vec{E}_0 e^{i(kna-\omega t)}}_{\vec{E}(na,t)}$

u_n 的解：$u_n(t) = \dfrac{\beta e^{i(kna-\omega t)}}{\omega_0^2-\omega^2}$

②偶极辐射

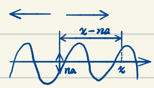
$\vec{E}_n(x,t)$ 第 n 个偶极子发射的电磁波

波动方程 $\partial_t^2 \vec{E}_n \approx v^2\partial_x^2 \vec{E}_n + \vec{j} = v^2 \partial_x^2 \vec{E}_n + q\ddot{\vec{u}}_n \delta(x-na)$

$\vec{E}_n(x,t)$ 的解 $\vec{E}_n(x,t) \propto \vec{E}_n^+(x,t) + \vec{E}_n^-(x,t)$ (通解 $f(x+vt)+g(x-vt)$)

$\vec{E}_n^\pm(x,t) \propto e^{i[\underbrace{\pm k(x-na)}_{①} - \omega t + \underbrace{kna}_{②} + \underbrace{\pi}_{③}]}$

- ① ± ~ 左/右行波
  $(x-na)$ ~ 从波源 $(x=na)$ 发出的波的相位积累



- ② 继承自源点处偶极振荡 u_n 的自带相位
- ③ $\ddot{\vec{u}}_n$ 相对于 $\vec{u}_n$ 的相位延迟 $\ddot{\vec{u}}_n \propto -e^{i(kna-\omega t)} = e^{i(kna-\omega t+\pi)}$

③干涉

前屏上的干涉 $(x=x_1)$：右行波的相加

$E_{前}(x_1,t) = \sum_n E_n^+(x_1,t) = \sum_n E_0 e^{i[k(x_1-na)-\omega t+kna+\pi]} = E_0\sum_n e^{i(kx_1-\omega t+\pi)} = nE_0 e^{i(kx_1-\omega t+\pi)}$

⇒ n 个子波均同相，相长干涉

后屏上的干涉 $(x=x_0)$：左行波的相加

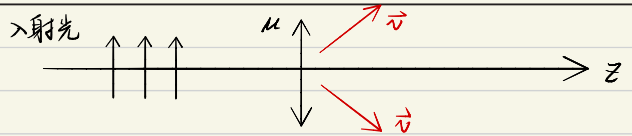
$E_{后}(x_2,t) = \sum_n E_n^-(x_2,t) = \sum_n E_0 e^{i[-k(x_2-na)-\omega t+kna+\pi]} = E_0 e^{i(-kx_2-\omega t+\pi)}\sum_n e^{i2kna}$

⇒ n 个子波，相邻者相位相差 2kna

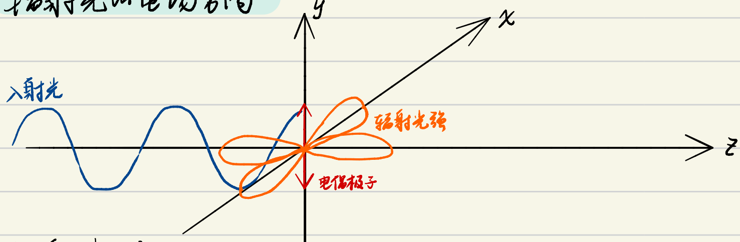
散射起偏

散射的基本物理过程

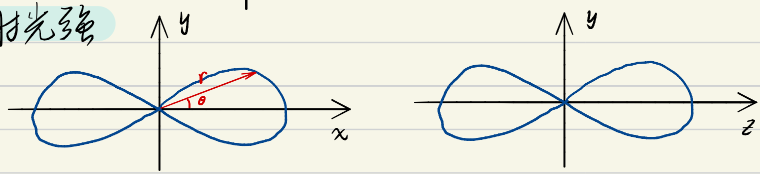
- 过程① 入射光激发散射子(电偶子)的振荡 ⇒ 偶极激发
- 过程② 电偶子激发后会向四周发射次级波 ⇒ 偶极辐射



辐射光的电场方向



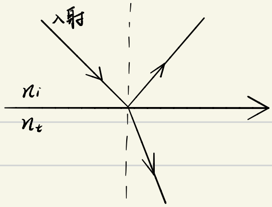
辐射光强

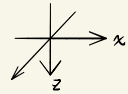


r : 光的强度
θ : 光的传播方向

散射光传播方向 / 入射光偏振方向	x 方向	y 方向	z 方向
y - 偏振	有散射，散射光偏振方向为 y 方向	无散射	有散射 y 偏振
x - 偏振	无散射	有散射 x 偏振	有散射 x 偏振
自然光 (有 x、y - 偏振)	有散射 y 偏振	有散射 x 偏振	有散射 偏振有 x、y 方向
	y - 线偏光	x - 线偏光	自然光

光的传播





入射面为 x, z 平面 ($k_i^y = 0$)

折射和反射

物理系统

已知：入射光的波函数

$$\vec{E}_i = \vec{E}_i^0 e^{i(\vec{k}_i\cdot\vec{r}-\omega_i t)} = (E_i^{0x}, E_i^{0y}, E_i^{0z})\, e^{i(k_i^x x + k_i^z z - \omega_i t)}$$

求：反射光的波函数

$$\vec{E}_r = \vec{E}_r^0 e^{i(\vec{k}_r\cdot\vec{r}-\omega_r t)} = (E_r^{0x}, E_r^{0y}, E_r^{0z})\, e^{i(k_r^x x + k_r^z z - \omega_r t)}$$

折射光的波函数

$$\vec{E}_t = \vec{E}_t^0 e^{i(\vec{k}_t\cdot\vec{r}-\omega_t t)} = (E_t^{0x}, E_t^{0y}, E_t^{0z})\, e^{i(k_t^x x + k_t^z z - \omega_t t)}$$

参数个数

入射光：$(\vec{E}_i^0, \vec{k}_i, \omega_i) = (E_i^{0x}, E_i^{0y}, E_i^{0z}, k_i^x, k_i^y = 0, k_i^z, \omega_i)$

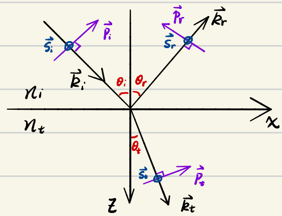
总参数 6　　自由参数 4

反射/折射光：$(\vec{E}_r^0, \vec{k}_r, \omega_r)$, $(\vec{E}_t^0, \vec{k}_t, \omega_t)$

总参数 7　　自由参数 5 (不预设 $k_r^y = 0$, $k_t^y = 0$)

已知&未知参数的优化

引入入射光和反射光的随动二维坐标系，该坐标系垂直于相应的波矢，由坐标轴 $(\hat{s}, \hat{p}_{i/r/t})$ 张成



s : senkrecht (垂直)

p : parallel (平行)

$$\hat{p}\times\hat{s} = \hat{k}$$

自由参数的选取

入射光：$(\omega_i, \theta_i, E_i^{0y}, E_i^{0p})$

$$\Rightarrow \left(\omega_i,\ k_i^x = \frac{\omega_i n_i}{c}\sin\theta_i,\ k_i^z = \frac{\omega_i n_i}{c}\cos\theta_i,\ E_i^{0x} = E_i^p\cos\theta_i,\ E_i^{0y},\ E_i^{0z} = -E_i^p\sin\theta_i\right)$$

反射光：$(\omega_r, \theta_r, E_r^{0y}, E_r^{0p}, k_r^y)$

$$\Rightarrow \left(\omega_r,\ k_r^x = \frac{\omega_r n_r}{c}\sin\theta_r,\ k_r^y = 0,\ k_r^z = -\frac{\omega_r n_r}{c}\cos\theta_r,\ E_r^{0x} = -E_r^p\cos\theta_r,\ E_r^{0y},\ E_r^{0z} = -E_r^p\sin\theta_r\right)$$

折射光：$(\omega_t, \theta_t, E_t^{0y}, E_t^{0p}, k_t^y)$

$$\Rightarrow \left(\omega_t,\ k_t^x = \frac{\omega_t n_t}{c}\sin\theta_t,\ k_t^y = 0,\ k_t^z = \frac{\omega_t n_t}{c}\cos\theta_t,\ E_t^{0x} = E_t^p\cos\theta_t,\ E_t^{0y},\ E_t^{0z} = -E_t^p\sin\theta_t\right)$$